The Social Value of Information in Cooperation*

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Abstract

This paper studies the social value of public information in cooperative settings. It establishes a novel link between the welfare effects of public information and its potential to destabilize socially desirable cooperative arrangements. When public signals do not destabilize any cooperative arrangement that is stable without them, public information cannot decrease welfare. In transferable utility games where stability is defined via the core, public information has no impact on welfare. By contrast, when public signals alter which cooperative arrangements are stable, public information can reduce welfare. Departing from the traditional focus on risk-sharing, we show that the social value of public information can be negative even when individuals are risk neutral. The underlying source of this welfare loss is a fundamental tension between efficiency and stability.

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1. Introduction

A seminal result in information economics is that public signals about the state of the economy can reduce welfare. Dating back to Hirshleifer (1971) and Wilson (1978), the core insight is that public information about payoff-relevant fundamentals can induce individuals to reject trades in contingent claims, thereby undermining ex-ante mutually beneficial risk pooling. This paper shows that the welfare costs of public information are not confined to environments where gains from risk-sharing are present. Even without risk aversion, revealing information about the state of the world can destabilize otherwise efficient cooperative outcomes. At its core, the problem is that public information can create a wedge between efficiency and stability in cooperative arrangements.[1]

This shift in focus—from risk-sharing to the broader notion of cooperation—allows us to establish a new result: public information can reduce welfare only if public signals destabilize some cooperative arrangement that is stable without them. Classical examples include Hirshleifer (1971) and Wilson (1978), where this destabilizing effect arises from agents' inability to make side payments. Moreover, we show that in transferable utility games where stability is defined via the core, the social value of public information is zero.

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[1]Our analysis is distinct from the literature initiated by Morris and Shin (2002), which studies the social value of public information when agents act individually but have a coordination motive, and receive imprecise public and private signals. In our setting, agents form coalitions, and public information matters because it affects which cooperative arrangements are stable.

To understand this result, consider a bargaining problem where individuals can make side payments, and the outcomes achievable by any coalition of players depend on a random event—the state of the world. If individuals bargain *before* the state is realized, an allocation is a rule assigning payoffs to players as a function of the realized state. Such a rule is in the ex-ante core if (i) in every state it exhausts the surplus of the grand coalition, and (ii) no coalition is assigned less than its expected worth. If individuals bargain *after* the state is realized, an allocation is in the (ex-post) core if it exhausts the surplus of the grand coalition and is not blocked by any other coalition in that state. Thus, regardless of whether individuals bargain before or after the state is realized, every core allocation distributes the entire surplus of the grand coalition. Public signals may shift individuals' bargaining positions, and hence how the surplus is divided, but they do not alter its total. Since the expected surplus of the grand coalition is the same whether or not the state is revealed, revealing the state of the world has no effect on expected welfare.

An important contribution of our paper is to demonstrate that public information can reduce welfare even in the absence of gains from risk-sharing. To illustrate this, we consider an example inspired by the peer-effects literature. Two risk-neutral workers face idiosyncratic productivity shocks and must decide whether to work separately or co-locate. When co-located, peer effects arise, and each worker's productivity is affected by that of her co-worker. In Hirshleifer (1971) and Wilson (1978), the tension between efficiency and stability arises from risk aversion coupled with the inability of agents to make side payments. In this example, the role of risk aversion is replaced by the assumption that peer effects are nonlinear. Specifically, the benefit a worker gains from being paired with a more productive peer exceeds the cost borne by the latter, making cooperation efficient when exactly one worker experiences a negative productivity shock. When side payments cannot be used, revealing information about workers' productivity shocks renders co-location unsustainable in this contingency, reducing expected output and welfare—even though the workers are risk neutral.

This paper makes three contributions. First, it shows that the negative social value of public information is not inherently due to diminished risk-sharing opportunities, but instead stems from a broader tension between efficiency and the stability of cooperative arrangements. In doing so, the paper connects the literature on the social value of information to the general problems of coalition formation and bargaining (e.g., Peleg and Sudhölter 2007; Ray 2007).[2] Second, it shows that public information can decrease welfare only if public signals destabilize some cooperative arrangement that is stable without them. As a benchmark, when utility is transferable and stability is defined via the core, public signals have no effect on welfare: with side payments, every core allocation exhausts the worth of the grand coalition in each state, so public information may change how this worth is divided, but not its expected total.[3] This role of side payments resembles that in Forges et al. (2002): there, they help satisfy incentive constraints under private information; here, they ensure that stability does not come at the expense of efficiency.[4] Finally, the paper develops several tractable examples showing that public information can increase or decrease welfare when it alters which cooperative arrangements are stable.

[2]Our analysis also relates to work on endogenous communication networks (e.g., Calvó-Armengol et al. 2015), where agents choose with whom to exchange information; here, information is public, and it shapes which coalitions are stable.

[3]This result is conceptually related to the Second Welfare Theorem: just as lump-sum transfers allow any Pareto efficient allocation to be decentralized as a competitive equilibrium, side payments allow the grand coalition to sustain an efficient arrangement after every signal realization.

[4]Our setting differs in that information is public, so that no incentive constraints arise, and the question is not how to elicit private information but whether acquiring public information raises or lowers welfare.

2. Model

Let N denote a finite set of players, with typical elements i, j. A coalition S is a nonempty subset of N, and a coalition structure $\mathcal{S}$ is a partition of N. Given a coalition structure $\mathcal{S}$, let $S(i)$ be the coalition in $\mathcal{S}$ that contains player i. The set N is referred to as the grand coalition.

Let Θ be a finite[5] set of possible states of the world, with typical element θ. The true state of the world is unknown to the players at the time they form cooperative arrangements. The players share a common prior belief over the states of the world, denoted by $\mu \in \Delta(\Theta)$, where $\Delta(\Theta)$ denotes the set of probability distributions over Θ.

Let X be a set of outcomes. We denote by $F(S, \theta) \subseteq X$ the set of outcomes that are feasible for coalition S in state θ. Given a coalition structure $\mathcal{S}$, an allocation is a function that assigns an outcome to each coalition $S \in \mathcal{S}$ and state θ. An allocation is feasible if the outcome assigned to each coalition $S \in \mathcal{S}$ belongs to the set of outcomes that are feasible for S in state θ.

Definition 1 (Allocation)**.** *An allocation on a coalition structure $\mathcal{S}$ is a function $x^{\mathcal{S}} : \mathcal{S} \times \Theta \to X$ that assigns to each coalition $S \in \mathcal{S}$ and state $\theta \in \Theta$ an outcome in X. An allocation $x^{\mathcal{S}}$ on $\mathcal{S}$ is feasible if, for each coalition $S \in \mathcal{S}$ and state $\theta \in \Theta$,*

$$x^{\mathcal{S}}(S, \theta) \in F(S, \theta).$$

Definition 2 (Cooperative arrangement)**.** *Let a cooperative arrangement be a pair $\left(\mathcal{S}, x^{\mathcal{S}}\right)$, where $\mathcal{S}$ is a coalition structure and $x^{\mathcal{S}}$ is an allocation on $\mathcal{S}$. A cooperative arrangement is feasible if its allocation $x^{\mathcal{S}}$ is feasible. We denote by CA the set of feasible cooperative arrangements.*

Each player i has preferences over outcomes represented by a von Neumann–Morgenstern utility function $u_i : X \to \mathbb{R}$. A game is a tuple $\Gamma = (N, \Theta, \mu, X, F, \boldsymbol{u})$, where $\boldsymbol{u} = (u_i)_{i \in N}$. An allocation is ex-ante blocked if there exists a coalition that can implement an alternative allocation giving each of its members a strictly higher expected payoff.

Definition 3 (Ex-ante blocked allocation)**.** *An allocation $x^{\mathcal{S}}$ is ex-ante blocked if there is a coalition $T \subseteq N$ and a function $y : \Theta \to X$ such that*

- *$y(\theta) \in F(T, \theta)$, for each $\theta \in \Theta$;*
- *$\sum_{\theta \in \Theta} \mu(\theta)\, u_i(y(\theta)) > \sum_{\theta \in \Theta} \mu(\theta)\, u_i\left(x^{\mathcal{S}}(S(i), \theta)\right)$, for each $i \in T$.*

Definition 4 (Ex-ante stable cooperative arrangement)**.** *A feasible cooperative arrangement $\left(\mathcal{S}, x^{\mathcal{S}}\right)$ is ex-ante stable if $x^{\mathcal{S}}$ is not ex-ante blocked. We denote by $\mathsf{SCA}^0 \subseteq \mathsf{CA}$ the set of ex-ante stable cooperative arrangements. We say that a coalition structure $\mathcal{S}$ is ex-ante stable if there exists an allocation $x^{\mathcal{S}}$ on $\mathcal{S}$ such that $\left(\mathcal{S}, x^{\mathcal{S}}\right) \in \mathsf{SCA}^0$.*

To analyze the social value of public information in cooperative settings, we follow Hirshleifer (1971) and we measure the social value of public information by comparing *utilitarian* welfare, evaluated from an ex-ante point of view, with and without a public signal. This comparison can be interpreted as the

[5] We assume that Θ is finite to avoid measure-theoretic complications. The main results extend to general state spaces under standard measurability assumptions, replacing sums with integrals.

problem of a utilitarian planner who shares the players' prior beliefs and decides whether to draw a public signal about the state of the world before play begins. Let Σ be a finite set of possible signal realizations and $\sigma \in \Sigma$ a typical realization. Let $\varsigma : \Theta \to \Delta(\Sigma)$ be a function assigning to each state θ a distribution over Σ; for each $\theta \in \Theta$ and $\sigma \in \Sigma$, we write $\varsigma(\sigma \mid \theta)$ for the probability that $\varsigma(\theta)$ assigns to σ. Let $\pi \in \Delta(\Sigma)$ be the induced marginal distribution over signal realizations, given by

$$\pi(\sigma) = \sum_{\theta \in \Theta} \mu(\theta)\, \varsigma(\sigma \mid \theta), \tag{1}$$

for each $\sigma \in \Sigma$. If a signal is drawn, players update their prior and form a cooperative arrangement conditional on the signal realization σ. Otherwise, they form a cooperative arrangement based on their prior. Upon observing a signal realization σ with $\pi(\sigma) > 0$, beliefs are updated according to Bayes' rule:

$$\mu(\theta \mid \sigma) = \frac{\mu(\theta)\, \varsigma(\sigma \mid \theta)}{\sum_{\theta' \in \Theta} \mu(\theta')\, \varsigma(\sigma \mid \theta')}.$$

Let $\Sigma^+ = \{\sigma \in \Sigma : \pi(\sigma) > 0\}$ denote the set of signal realizations with positive probability. By Bayes' rule and Equation (1), $\pi(\sigma)\, \mu(\theta \mid \sigma) = \mu(\theta)\, \varsigma(\sigma \mid \theta)$ for each $\sigma \in \Sigma^+$ and $\theta \in \Theta$. Moreover, if $\pi(\sigma) = 0$, then $\mu(\theta)\, \varsigma(\sigma \mid \theta) = 0$ for each $\theta \in \Theta$. Hence, for each $\theta \in \Theta$,

$$\sum_{\sigma \in \Sigma^+} \pi(\sigma)\, \mu(\theta \mid \sigma) = \sum_{\sigma \in \Sigma} \mu(\theta)\, \varsigma(\sigma \mid \theta) = \mu(\theta). \tag{2}$$

If a signal is drawn and realization $\sigma \in \Sigma^+$ is observed, the players' prior μ is replaced by the posterior $\mu(\cdot \mid \sigma)$, and game $\Gamma(\sigma) = (N, \Theta, \mu(\cdot \mid \sigma), X, F, \boldsymbol{u})$ is played instead of Γ. Definitions 3 and 4 extend to $\Gamma(\sigma)$ by replacing μ with $\mu(\cdot \mid \sigma)$:

Definition 5 (Allocation blocked under signal realization σ)**.** *Fix a signal realization $\sigma \in \Sigma^+$. An allocation $x^{\mathcal{S}}$ is blocked under σ if there is a coalition $T \subseteq N$ and a function $y : \Theta \to X$ such that*

- *$y(\theta) \in F(T, \theta)$, for each $\theta \in \Theta$;*
- *$\sum_{\theta \in \Theta} \mu(\theta \mid \sigma)\, u_i(y(\theta)) > \sum_{\theta \in \Theta} \mu(\theta \mid \sigma)\, u_i\left(x^{\mathcal{S}}(S(i), \theta)\right)$, for each $i \in T$.*

Definition 6 (Cooperative arrangement stable under signal realization σ)**.** *Fix a signal realization $\sigma \in \Sigma^+$. A feasible cooperative arrangement $\left(\mathcal{S}, x^{\mathcal{S}}\right)$ is stable under σ if $x^{\mathcal{S}}$ is not blocked under signal realization σ. We denote by $\mathsf{SCA}^\sigma \subseteq \mathsf{CA}$ the set of cooperative arrangements stable under signal realization σ. We say that a coalition structure $\mathcal{S}$ is stable under signal realization σ if there exists an allocation $x^{\mathcal{S}}$ on $\mathcal{S}$ such that $\left(\mathcal{S}, x^{\mathcal{S}}\right) \in \mathsf{SCA}^\sigma$.*

For each feasible cooperative arrangement $\left(\mathcal{S}, x^{\mathcal{S}}\right) \in \mathsf{CA}$ and each belief $\nu \in \Delta(\Theta)$, let

$$W\left(\mathcal{S}, x^{\mathcal{S}}; \nu\right) = \sum_{i \in N} \sum_{\theta \in \Theta} \nu(\theta)\, u_i\left(x^{\mathcal{S}}(S(i), \theta)\right)$$

denote the total expected welfare of $\left(\mathcal{S}, x^{\mathcal{S}}\right)$ under belief ν.

When several cooperative arrangements are stable, stability alone does not determine which one forms, so welfare is not uniquely defined. To compare welfare with and without a signal, we evaluate it at

the best stable cooperative arrangement, that is, the one that maximizes total expected welfare among all stable arrangements, both without a signal and after each signal realization. Let

$$W^0 = \max_{(\mathcal{S},x^{\mathcal{S}})\in \mathsf{SCA}^0} W\left(\mathcal{S}, x^{\mathcal{S}}; \mu\right)$$

denote welfare without a signal and, for each $\sigma \in \Sigma^+$, let

$$W^\sigma = \max_{(\mathcal{S},x^{\mathcal{S}})\in \mathsf{SCA}^\sigma} W\left(\mathcal{S}, x^{\mathcal{S}}; \mu(\cdot \mid \sigma)\right)$$

denote welfare under signal realization σ. The social value of public information is $\sum_{\sigma\in\Sigma^+} \pi(\sigma)\, W^\sigma - W^0$. We assume that the maxima defining W^0 and W^σ exist. This holds in two cases of interest. First, in pure hedonic games (Banerjee et al. 2001; Bogomolnaia and Jackson 2002) with finitely many players and states, the set of feasible cooperative arrangements is finite, so the maxima exist whenever stable cooperative arrangements exist. Second, in transferable utility games where stability is defined via the core, every stable allocation exhausts the worth of the grand coalition. Hence, if the core is nonempty with and without a signal, welfare equals the expected worth of the grand coalition under the relevant belief: the prior or the posterior (see Subsection 2.2).

The following theorem shows that when drawing a signal does not destabilize any cooperative arrangement that is stable without it, public information cannot reduce ex-ante welfare. Intuitively, a public signal does not alter the outcomes feasible for each coalition in each state. As a consequence, public information can affect ex-ante welfare only if it leads the players to form a different cooperative arrangement—either a different partition or a different allocation—than they would without signals.

Theorem 1. *Suppose that* $\mathsf{SCA}^0 \subseteq \mathsf{SCA}^\sigma$ *for each signal realization* $\sigma \in \Sigma^+$. *Then,*

$$W^0 \leq \sum_{\sigma\in\Sigma^+} \pi(\sigma)\, W^\sigma.$$

Proof. Let $\left(\mathcal{S}, x^{\mathcal{S}}\right) \in \mathsf{SCA}^0$ be a cooperative arrangement that attains W^0, that is,

$$\sum_{i\in N}\sum_{\theta\in\Theta} \mu(\theta)\, u_i\left(x^{\mathcal{S}}\left(S(i),\theta\right)\right) = W^0.$$

By assumption, $\left(\mathcal{S}, x^{\mathcal{S}}\right) \in \mathsf{SCA}^\sigma$ for each $\sigma \in \Sigma^+$. Hence, for each $\sigma \in \Sigma^+$,

$$W^\sigma \geq \sum_{i\in N}\sum_{\theta\in\Theta} \mu(\theta \mid \sigma)\, u_i\left(x^{\mathcal{S}}\left(S(i),\theta\right)\right).$$

Multiplying both sides by $\pi(\sigma)$, summing over $\sigma \in \Sigma^+$, and using Equation (2),

$$\begin{aligned}\sum_{\sigma\in\Sigma^+} \pi(\sigma)\, W^\sigma &\geq \sum_{i\in N}\sum_{\theta\in\Theta}\left(\sum_{\sigma\in\Sigma^+} \pi(\sigma)\,\mu(\theta \mid \sigma)\right) u_i\left(x^{\mathcal{S}}\left(S(i),\theta\right)\right)\\ &= \sum_{i\in N}\sum_{\theta\in\Theta} \mu(\theta)\, u_i\left(x^{\mathcal{S}}\left(S(i),\theta\right)\right) = W^0.\end{aligned}$$

□

Theorem 1 evaluates welfare at the best stable cooperative arrangement, both with and without a signal; that is, it measures welfare by the highest total expected welfare attainable subject to stability, without taking a stand on which stable arrangement forms. This choice matters for the result.[6] To see this, suppose that every cooperative arrangement that is stable without a signal remains stable after each signal realization, as in Theorem 1. If welfare is instead evaluated at the worst stable arrangement, the same argument, applied in reverse, shows that public information cannot increase welfare. If the same stable arrangement is selected whether or not a signal is drawn, public information has no effect on welfare, by the law of iterated expectations. Hence, when public information does not destabilize any cooperative arrangement, its welfare effect is driven entirely by how an arrangement is selected among the stable ones. Welfare effects that do not depend on the selection rule must therefore arise from destabilization. See Subsection 2.4 for additional details.

2.1. Stability and the welfare effect of public information

The inequality in Theorem 1 can be strict. Even when public information leaves the set of stable cooperative arrangements unchanged, it can increase welfare as measured by the best stable arrangement. The reason is that the best stable arrangement may differ across signal realizations. Without a signal, welfare is evaluated at a single arrangement, the best on average across states. With a signal, it is evaluated at the best stable arrangement given each signal realization. The following example illustrates this in a pure hedonic game, where each coalition has a single feasible outcome in each state, so that the welfare gain does not stem from the allocation of resources within coalitions.

2.1.1. Example: A welfare gain from public information with unchanged stable arrangements

Let $N = \{1, 2, 3\}$ and $\Theta = \{\theta_1, \theta_2\}$, with $\mu(\theta_1) = \mu(\theta_2) = 1/2$. The planner can draw a signal that perfectly reveals the state. Each coalition has a single feasible outcome in each state, and payoffs are as follows. Every player obtains 0 alone, and every member of coalitions $\{1, 3\}$ and N obtains -10 in both states. Player 2 obtains 1 in coalition $\{1, 2\}$ and in coalition $\{2, 3\}$, in both states. Player 1 obtains 3 in coalition $\{1, 2\}$ in state θ_1 and 1 in state θ_2. Player 3 obtains 1 in coalition $\{2, 3\}$ in state θ_1 and 3 in state θ_2. Table 1 reports expected payoffs without a signal and after each signal realization.

	No signal	Signal: θ_1	Signal: θ_2
Player 1 in $\{1, 2\}$	2	3	1
Player 3 in $\{2, 3\}$	2	1	3
Player 2 in $\{1, 2\}$ or $\{2, 3\}$	1	1	1
Any player alone	0	0	0
Any member of $\{1, 3\}$ or N	-10	-10	-10

Table 1: Expected payoffs without a signal and after each signal realization.

Consider the coalition structures $\mathcal{S}_1 = \{\{1, 2\}, \{3\}\}$ and $\mathcal{S}_2 = \{\{2, 3\}, \{1\}\}$. Coalition structure $\mathcal{S}_1$ is stable without a signal and after each signal realization. Player 1 and player 2 would obtain 0 alone, strictly less than in $\{1, 2\}$. Player 3 would obtain 0 alone, which is not strictly more than what she obtains in $\mathcal{S}_1$. Coalition $\{2, 3\}$ cannot block, because player 2 obtains 1 in both $\{1, 2\}$ and $\{2, 3\}$. Coalitions

[6] Other evaluation criteria are conceivable. For instance, the criterion most favorable to public information evaluates welfare at the best stable cooperative arrangement when a signal is drawn and at the worst stable cooperative arrangement without a signal; the criterion least favorable to public information does the opposite.

$\{1,3\}$ and N cannot block, because their members obtain -10. By symmetry, $\mathcal{S}_2$ is also stable without a signal and after each signal realization. Every other coalition structure is blocked in each case: the structure in which all players are alone is blocked by $\{1,2\}$, and any structure containing $\{1,3\}$ or N is blocked by a player who leaves to be alone. Since each coalition structure admits a single feasible allocation, $\mathsf{SCA}^0 = \mathsf{SCA}^\sigma$ for each signal realization σ. Without a signal, both $\mathcal{S}_1$ and $\mathcal{S}_2$ yield total expected welfare $1+2=3$, so $W^0 = 3$. With a signal, the best stable arrangement is $\mathcal{S}_1$ when the signal reveals θ_1, and $\mathcal{S}_2$ when it reveals θ_2. Each yields total welfare $1+3=4$. Hence,

$$\sum_{\sigma \in \Sigma^+} \pi(\sigma)\, W^\sigma = 4 > 3 = W^0.$$

The assumption of Theorem 1 is stated in terms of cooperative arrangements, that is, coalition structures together with their allocations, rather than in terms of coalition structures alone. A condition on coalition structures alone would not suffice. Public information may leave the set of stable coalition structures unchanged while destabilizing the allocation that a coalition uses to share its surplus, and this alone can reduce welfare. The following example illustrates this.[7]

2.1.2. Example: A welfare loss from public information with an unchanged stable coalition structure

Let $N = \{1,2\}$ and $\Theta = \{\theta_1, \theta_2\}$, with $\mu(\theta_1) = \mu(\theta_2) = 1/2$. The planner can draw a signal that perfectly reveals the state. Outcomes are payoff profiles, and each player's utility is her own payoff. Each player obtains 0 alone. In state θ_1, the grand coalition can choose either $(10,-1)$ or $(\varepsilon,\varepsilon)$, where $0 < \varepsilon < 1$. In state θ_2, it can choose either $(-1,10)$ or $(\varepsilon,\varepsilon)$. Without a signal, consider the allocation on the grand coalition that assigns $(10,-1)$ in state θ_1 and $(-1,10)$ in state θ_2. Each player obtains an expected payoff of $(10-1)/2 = 4.5 > 0$, so no player can block by leaving. Nor can the grand coalition block: total payoff is at most 9 in each state, so no allocation gives both players more than 4.5 in expectation. This allocation is therefore ex-ante stable, and it yields total expected welfare 9. The coalition structure in which both players are alone is blocked by the grand coalition choosing $(\varepsilon,\varepsilon)$ in both states. Hence, the grand coalition is the only ex-ante stable coalition structure, and $W^0 = 9$.

With a signal, suppose it reveals θ_1. Any allocation on the grand coalition that assigns $(10,-1)$ in state θ_1 is blocked by player 2, who obtains 0 alone. Any allocation that assigns $(\varepsilon,\varepsilon)$ in state θ_1 is stable: no player gains by leaving, since $\varepsilon > 0$, and the grand coalition cannot block, since switching to $(10,-1)$ makes player 2 worse off. The coalition structure in which both players are alone is still blocked. The case in which the signal reveals θ_2 is symmetric. Hence, the grand coalition remains the only stable coalition structure after each signal realization, but $W^\sigma = 2\varepsilon$ for each σ, so that

$$\sum_{\sigma \in \Sigma^+} \pi(\sigma)\, W^\sigma = 2\varepsilon < 9 = W^0.$$

The set of stable coalition structures is unchanged, yet public information reduces welfare, because it destabilizes the state-contingent allocation that the grand coalition uses without a signal. In terms of Theorem 1, this allocation belongs to SCA^0 but not to SCA^σ, so the assumption of the theorem fails.

The problem illustrated by this example cannot arise in pure hedonic games (Banerjee et al. 2001;

[7] We thank an anonymous referee for suggesting this example.

Bogomolnaia and Jackson 2002), where each coalition has a single feasible outcome in each state. There, each coalition structure admits a unique feasible allocation, so public information cannot change which allocations are stable without changing which coalition structures are stable. Hence, in pure hedonic games, the assumption of Theorem 1 reduces to a condition on coalition structures alone: if every coalition structure that is stable without a signal remains stable after each signal realization, public information cannot reduce welfare.[8]

2.2. *Transferable utility*

In this subsection, we examine a cooperative game where utility is transferable and stability is defined via the core, implying that only the grand coalition can be a stable coalition structure, regardless of whether the planner draws a signal about the state of the world. In this setting, public information is welfare neutral. The reason is that every core allocation distributes the entire worth of the grand coalition among the players. Public information may change how this worth is divided, but not its expected total.

Transferable utility arises, for example, when players have quasilinear preferences over a commodity they can exchange freely (i.e., "money"). Then, the cooperative possibilities of a game can be described by a value (or characteristic) function v, which assigns a number $v(S,\theta)$ to each coalition S and state θ. Here, $v(S,\theta)$ represents the total transferable utility that coalition S can generate when the state of the world is θ (i.e., the "worth" of coalition S in state θ). Formally, a value function is a mapping

$$v : \left(2^N \setminus \{\emptyset\}\right) \times \Theta \to \mathbb{R}.$$

This setting is a special case of the model in Section 2, in which outcomes are payoff profiles: $X = \mathbb{R}^N$, $u_i(z) = z_i$ for each $i \in N$ and $z \in X$, and $F(S,\theta) = \{z \in \mathbb{R}^N : \sum_{j\in S} z_j \leq v(S,\theta)\}$ for each coalition S and state θ. Following the tradition in TU games, we focus on the grand coalition and define stability via the core. An allocation is then a function $x : \Theta \to \mathbb{R}^N$, where $x(\theta)$ is an assignment of *realized* payoffs to the players when the state of the world is θ, and W^0 and W^σ denote total expected welfare at a core allocation without a signal and under signal realization $\sigma \in \Sigma^+$, respectively.

Definition 7. *An allocation x is in the ex-ante core if it exhausts the worth of the grand coalition in every state, that is, $\sum_{i\in N} x_i(\theta) = v(N,\theta)$ for each $\theta \in \Theta$—in which case it is said to be efficient*[9]*—and is not ex-ante blocked; i.e., there exists no coalition S and function $y : \Theta \to \mathbb{R}^S$ such that*

$$\sum_{j\in S} y_j(\theta) \leq v(S,\theta), \quad \textit{for each } \theta \in \Theta,$$

[8] A condition on coalition structures alone suffices whenever stability does not force coalitions to waste surplus. Suppose that every coalition structure that is stable without a signal remains stable after each signal realization $\sigma \in \Sigma^+$, and that every coalition structure $\mathcal{S}$ that is stable under σ admits an allocation $t^{\mathcal{S}}_\sigma$ such that $(\mathcal{S}, t^{\mathcal{S}}_\sigma) \in \mathsf{SCA}^\sigma$ and $\sum_{i\in S} u_i(t^{\mathcal{S}}_\sigma(S,\theta)) \geq \sum_{i\in S} u_i(z)$ for each coalition $S \in \mathcal{S}$, state $\theta \in \Theta$, and outcome $z \in F(S,\theta)$. Then $W^0 \leq \sum_{\sigma\in\Sigma^+} \pi(\sigma) W^\sigma$. Indeed, if $(\mathcal{S}, x^{\mathcal{S}})$ attains W^0, then $W^\sigma \geq W(\mathcal{S}, t^{\mathcal{S}}_\sigma; \mu(\cdot \mid \sigma)) \geq W(\mathcal{S}, x^{\mathcal{S}}; \mu(\cdot \mid \sigma))$ for each $\sigma \in \Sigma^+$, and the result follows from Equation (2), as in the proof of Theorem 1. Pure hedonic games satisfy this condition trivially, whereas the example in Subsection 2.1.2 violates it.

[9] For this terminology to be meaningful, it must be the case that the worth of the grand coalition always exceeds that of any other coalition structure. A natural adaptation of the standard superadditivity assumption offers a justification for this: $v(S\cup T,\theta) \geq v(S,\theta) + v(T,\theta)$ for each state θ and disjoint coalitions S and T. A well-established practice is to invoke superadditivity to justify the assumption that players form the grand coalition and share its worth (Peleg and Sudhölter 2007; Myerson 2013).

and

$$\sum_{\theta\in\Theta}\mu(\theta)\sum_{j\in S}y_j\left(\theta\right)>\sum_{\theta\in\Theta}\mu(\theta)\sum_{j\in S}x_j\left(\theta\right).$$

We denote by Core *the set of allocations that satisfy these conditions in game* $\mathcal{G}=(N,\Theta,\mu,v)$.

The core is nonempty if and only if $\overline{v}$ is balanced, where $\overline{v}:2^N\setminus\{\emptyset\}\to\mathbb{R}$ is the function such that

$$\overline{v}(S)=\sum_{\theta\in\Theta}\mu(\theta)\,v(S,\theta),$$

for each coalition S. This is a direct application of the Bondareva–Shapley theorem.

If the planner draws a signal about the state of the world, the players' prior μ is replaced by the posterior $\mu(\cdot\mid\sigma)$, and game $\mathcal{G}\left(\sigma\right)=(N,\Theta,\mu(\cdot\mid\sigma),v)$ is played instead of $\mathcal{G}$. For each $\sigma\in\Sigma^+$, the core under signal realization σ is the set of efficient allocations that are not blocked under signal realization σ—meaning that there exists no coalition S and function $y:\Theta\to\mathbb{R}^S$ such that

$$\sum_{j\in S}y_j\left(\theta\right)\leq v\left(S,\theta\right),$$

for each $\theta\in\Theta$, and

$$\sum_{\theta\in\Theta}\mu(\theta\mid\sigma)\sum_{j\in S}y_j\left(\theta\right)>\sum_{\theta\in\Theta}\mu(\theta\mid\sigma)\sum_{j\in S}x_j\left(\theta\right).$$

We denote by $\mathrm{Core}\left(\sigma\right)$ the set of allocations that satisfy these conditions. For each $\sigma\in\Sigma^+$, $\mathrm{Core}\left(\sigma\right)$ is nonempty if and only if $\overline{v}^\sigma$ is balanced, where $\overline{v}^\sigma:2^N\setminus\{\emptyset\}\to\mathbb{R}$ is the function such that

$$\overline{v}^\sigma(S)=\sum_{\theta\in\Theta}\mu(\theta\mid\sigma)\,v(S,\theta),\tag{3}$$

for each coalition S. Balancedness of $\overline{v}^\sigma$ for each $\sigma\in\Sigma^+$ is sufficient to ensure that the ex-ante core is nonempty (see Claim 2).

Claim 1 shows that the planner derives no expected gain from drawing a public signal about the state. Intuitively, although more precise public information may shift individual bargaining positions—and thus the division of surplus—it does not alter the expected surplus generated by the grand coalition.

Claim 1. *Suppose that utility is transferable, stability is defined via the core, and* Core *and* $\mathrm{Core}(\sigma)$ *are nonempty for each signal realization* $\sigma\in\Sigma^+$. *Then, the social value of public information is zero.*

Proof. Every allocation in Core distributes the worth of the grand coalition in each state, so $W^0=\sum_{\theta\in\Theta}\mu(\theta)\,v(N,\theta)$, whichever core allocation forms. Similarly, for each $\sigma\in\Sigma^+$, every allocation in $\mathrm{Core}(\sigma)$ distributes $v(N,\theta)$ in each state $\theta\in\Theta$, so $W^\sigma=\sum_{\theta\in\Theta}\mu(\theta\mid\sigma)\,v(N,\theta)$. By Equation (2),

$$\sum_{\sigma\in\Sigma^+}\pi(\sigma)\,W^\sigma=\sum_{\sigma\in\Sigma^+}\pi(\sigma)\sum_{\theta\in\Theta}\mu(\theta\mid\sigma)\,v(N,\theta)=\sum_{\theta\in\Theta}\mu(\theta)\,v(N,\theta)=W^0.$$

□

As discussed above, the key insight is that every core allocation distributes the entire worth of the grand coalition among the players, with or without a signal. Public information may shift individual

bargaining positions, and thus the division of surplus, but it cannot change the total: in each case, welfare equals the expected worth of the grand coalition, and by the law of iterated expectations its expected value is the same with and without a signal.[10]

The following claim shows that balancedness of $\overline{v}^{\sigma}$, for each signal realization $\sigma \in \Sigma^{+}$, is a sufficient condition for the core of $\mathcal{G}$ to be nonempty.

Claim 2 (Sufficient condition for Core $\neq \emptyset$)**.** *For each $\sigma \in \Sigma^{+}$, let $\overline{v}^{\sigma} : 2^{N} \setminus \{\emptyset\} \to \mathbb{R}$ be the value function defined in Equation (3). Suppose that, for each signal realization $\sigma \in \Sigma^{+}$, $\overline{v}^{\sigma}$ is balanced. Then, $\mathrm{Core}(\sigma)$ is nonempty for each $\sigma \in \Sigma^{+}$. Moreover, the ex-ante core is nonempty.*

Proof. Since each $\overline{v}^{\sigma}$ is balanced, we have that, for every function $\alpha : 2^{N} \setminus \{\emptyset\} \to [0,1]$ such that

$$\sum_{S \in 2^{N} \setminus \{\emptyset\} : i \in S} \alpha(S) = 1, \quad \text{for each } i \in N,$$

the following inequality holds for each signal realization $\sigma \in \Sigma^{+}$:

$$\sum_{S \in 2^{N} \setminus \{\emptyset\}} \alpha(S)\, \overline{v}^{\sigma}(S) \leq \overline{v}^{\sigma}(N).$$

Multiplying both sides by $\pi(\sigma) \geq 0$ and summing over $\sigma \in \Sigma^{+}$ preserves the inequality, because nonnegative linear combinations preserve weak inequalities. Thus, we obtain:

$$\sum_{\sigma \in \Sigma^{+}} \pi(\sigma) \sum_{S \in 2^{N} \setminus \{\emptyset\}} \alpha(S)\, \overline{v}^{\sigma}(S) \leq \sum_{\sigma \in \Sigma^{+}} \pi(\sigma)\, \overline{v}^{\sigma}(N).$$

By Equation (2), we have:

$$\sum_{\sigma \in \Sigma^{+}} \pi(\sigma)\, \overline{v}^{\sigma}(S) = \sum_{\theta \in \Theta} \left(\sum_{\sigma \in \Sigma^{+}} \pi(\sigma)\, \mu(\theta \mid \sigma) \right) v(S, \theta) = \sum_{\theta \in \Theta} \mu(\theta)\, v(S, \theta) = \overline{v}(S),$$

for each coalition S. Thus,

$$\sum_{S \in 2^{N} \setminus \{\emptyset\}} \alpha(S)\, \overline{v}(S) \leq \overline{v}(N).$$

This shows that $\overline{v}$ satisfies the Bondareva–Shapley condition. □

2.3. Nontransferable utility

In this subsection, we present several examples illustrating the social value of public information in environments where public information generally alters the set of stable coalition structures, thereby affecting welfare. While Theorem 1 applies to any number of players and any signal structure, the examples in this subsection consider only two players and perfectly informative signals. This choice is intentional: the purpose is to illustrate how public information affects welfare under nontransferable utility

[10] Note that neutrality does not follow from the fact that the grand coalition is the only stable coalition structure in both cases. As the example in Subsection 2.1.2 shows, public information can reduce welfare even when the stable coalition structure is unchanged. Neutrality relies instead on transferable utility, which ensures that no core allocation leaves any of the grand coalition's worth unallocated.

by altering the set of stable cooperative arrangements (in these examples, coalition structures, since each coalition structure admits a unique feasible allocation).[11] Each example highlights a distinct mechanism: the first shows that public information can lower welfare by eliminating risk-sharing opportunities; the second shows that public signals can reduce welfare even when players are risk neutral; the third demonstrates cases in which public information improves welfare; and the fourth shows that the effect of public information depends on the fundamentals of the economy. Thus, these examples illustrate that when public information does destabilize some cooperative arrangement, the sign of the welfare effect depends on the details of the environment.

The first example (Subsection 2.3.1), inspired by Hirshleifer (1971) and Wilson (1978), shows how public information can undermine risk-sharing opportunities and decrease welfare. Two workers, subject to idiosyncratic productivity shocks, can form a syndicate to pool and equally share income. Ex ante, this arrangement is socially optimal, as it insures each worker against negative shocks. A planner chooses whether to publicly reveal the workers' productivity shocks before they decide whether to form the syndicate. Revealing the state strictly reduces expected welfare by making it impossible to transfer output from a lucky worker to an unlucky one. We also show that if players can use side payments to compensate each other for participating in a syndicate, revealing the state has no effect on welfare.[12]

The example in Subsection 2.3.2 shows that the negative social value of public information does not necessarily arise from forgone risk-sharing. To make this point, we consider a version of the example in Subsection 2.3.1 where the workers are risk neutral. As before, each worker faces an idiosyncratic productivity shock that may reduce her productivity. The workers can either work in separate rooms or share a room, in which case they generate peer effects. These effects make it beneficial for a worker who receives a negative shock to work with one who does not, but impose a cost on the latter. The role played by risk aversion in the example in Subsection 2.3.1 is here replaced by a nonlinearity in the peer effects. Specifically, the productivity gain for the unlucky worker from working with a lucky one exceeds the loss incurred by the latter. As a result, it is socially optimal ex ante for the workers to share a room. However, public information makes this arrangement unstable.

The example in Subsection 2.3.3 presents an environment in which public information unambiguously improves expected welfare. There are two risk-averse entrepreneurs who may jointly undertake a project that requires simultaneous participation from both. The project's return is uncertain at the time of the participation decision. Without information, the project may not be undertaken, missing gains from cooperation in states where it would have been efficient. Alternatively, the project may be undertaken, leading to losses in states where cooperation is inefficient. Public information strictly improves expected welfare by ensuring that the project is undertaken if and only if it is efficient to do so.[13]

The example in Subsection 2.3.4 presents an environment in which the welfare effect of public information depends on model parameters. Two individuals are each working on a project and can agree to "exchange favors," meaning each exerts effort to help the other. This help is costly and yields a benefit

[11] Partial revelation does not change the mechanism at work: Theorem 1 and Claim 1 hold for any signal structure, and a less informative signal pools some states, so that cooperative arrangements that full revelation would destabilize may remain stable after some signal realizations.

[12] Versions of the examples with n players can be constructed along the same lines (e.g., by letting n workers form a syndicate that pools output equally); we restrict attention to two players because the mechanism is already present in this case.

[13] The idea that incomplete information can unravel socially efficient trades echoes the "lemons" intuition of Akerlof (1970), but in a setting where uncertainty is *symmetric*: the "quality" of the project is unknown to both sides at the time of decision.

that depends on the uncertain quality of the partner's assistance. The welfare impact of public information about favor quality hinges on the cost of exchanging favors. When the cost is low (high), favors are always (never) exchanged, so public information is irrelevant. At intermediate costs, however, public information affects behavior and can raise or lower welfare. If the cost is low enough that individuals are willing to exchange favors when both favors are of high quality—but still high enough that the expected payoff of favor exchange under uncertainty is negative—public information improves welfare by enabling favor exchange in the right states. Conversely, if individuals are willing to exchange favors under uncertainty, but would not do so if they knew one favor was of low quality, then information can reduce welfare by blocking socially efficient exchanges.[14]

In each example, absent side payments, every coalition structure $\mathcal{S}$ admits a unique feasible allocation $x^{\mathcal{S}}$, and we write $W(\mathcal{S}) = W\left(\mathcal{S}, x^{\mathcal{S}}; \mu\right)$ for its total expected welfare under the prior.

2.3.1. Example: Syndication

Consider an economy inhabited by two workers, $N = \{1, 2\}$. Each worker's output, q_i, is determined by an idiosyncratic productivity shock. A state of the world is a productivity profile $\theta = (\theta_1, \theta_2) \in \Theta = \{L, H\}^2$, where $0 < L < H$, and worker i's output in state θ is $q_i = \theta_i$. Productivity shocks are independent across workers, and each worker's productivity is H with probability $p \in (0, 1)$; that is, $\mu(\theta) = p^{k(\theta)}(1-p)^{2-k(\theta)}$ for each $\theta \in \Theta$, where $k(\theta)$ is the number of workers whose productivity is H in state θ. Workers can form a syndicate, in which case they commit to pooling output and splitting it equally among themselves. Specifically, worker i's consumption is

$$c_i = \begin{cases} \dfrac{q_1 + q_2}{2} & \text{if they form the syndicate,} \\ q_i & \text{otherwise.} \end{cases}$$

Workers have preferences over consumption represented by a strictly concave vNM utility function φ. Using the notation introduced in Subsection 2.3, this game has two possible coalition structures, corresponding to autarky and syndication, respectively; i.e., $\{\{1\}, \{2\}\}$ and $\{\{1, 2\}\}$. Given a coalition structure $\mathcal{S}$, the only feasible allocation is given by the function $x : \mathcal{S} \times \Theta \to \mathbb{R}^2$ such that

$$x_i(S(i), \theta) = \begin{cases} \theta_i & \text{if } S(i) = \{i\} \\ \frac{\theta_i + \theta_j}{2} & \text{if } S(i) = \{1, 2\} \end{cases},$$

for each $i \in \{1, 2\}$, where $j \neq i$ denotes the other worker. Since each coalition structure $\mathcal{S}$ determines a unique feasible allocation, we denote the latter by $x^{\mathcal{S}}$, and write $x_i^{\mathcal{S}}(\theta)$ for the outcome assigned to player i in state θ. To simplify notation even further, denote coalition structures $\{\{1\}, \{2\}\}$ and $\{\{1, 2\}\}$ by no-coop and coop, respectively. The following claim shows that, from an ex-ante perspective, it is socially optimal for the workers to form the syndicate.[15]

Claim 3. *Expected welfare is maximized when the workers form a syndicate.*

[14] While the information prevents favor exchange in states where both favors are low quality (which is socially inefficient), it also blocks exchange in states where it is socially desirable.

[15] In fact, forming the syndicate is the only ex-ante Pareto optimal outcome in this example, since it gives each worker a strictly higher expected utility than autarky (see the proof of Claim 3).

Proof. Under syndication, each worker's expected utility is

$$p^2\varphi(H) + 2p(1-p)\,\varphi\left(\frac{H+L}{2}\right) + (1-p)^2\varphi(L),$$

and under autarky it is $p\,\varphi(H) + (1-p)\,\varphi(L)$. Hence,

$$W(\text{coop}) - W(\text{no-coop}) = 4p(1-p)\left[\varphi\left(\frac{H+L}{2}\right) - \frac{\varphi(H)+\varphi(L)}{2}\right].$$

Since φ is strictly concave, Jensen's inequality implies that this difference is strictly positive. □

The following claim shows that the only ex-ante stable coalition structure is syndication.

Claim 4. *The only ex-ante stable coalition structure is* coop.

Proof. Each worker's expected utility is $W(\text{coop})/2$ under syndication and $W(\text{no-coop})/2$ under autarky. By Claim 3, $W(\text{coop}) > W(\text{no-coop})$. Hence, no worker can block syndication by leaving, while autarky is blocked by $\{1,2\}$. □

Next, suppose a planner can draw a public signal about each worker's productivity shock, with realizations $\sigma = (\sigma_1, \sigma_2) \in \{\ell, h\}^2$, before they decide whether to form a syndicate. Since the signal is perfectly informative, $\sigma_i = \ell$ if and only if $\theta_i = L$, for each $i = 1, 2$. The expected social value of drawing a signal is negative. To see why, note that revealing the state leads to two cases: either both workers have the same productivity, or one is high and the other low. In the first case, they are indifferent about forming a syndicate. In the second, the high-productivity worker refuses to share income with the low-productivity one. Thus, signal realizations $\sigma \in \{(\ell, h), (h, \ell)\}$ prevent output sharing when productivities differ, which eliminates the possibility of mutually beneficial risk-sharing. The following claim formalizes this intuition.

Claim 5. *Drawing a perfectly informative signal about the state of the world decreases expected welfare.*

Proof. Suppose a signal is drawn. When $\sigma \in \{(\ell, h), (h, \ell)\}$, the only stable coalition structure is no-coop. On the other hand, if $\sigma \in \{(\ell, \ell), (h, h)\}$, the workers are indifferent between cooperating and not. Therefore, from an ex-ante perspective, a worker's expected utility when a signal is drawn is

$$p\varphi\left(H\right) + (1-p)\,\varphi\left(L\right).$$

Claim 3 shows that this expected utility is strictly lower than the expected utility without a signal. □

If workers could make side payments to redistribute the surplus from efficient cooperative arrangements, a worker with a low productivity realization could compensate one with a high realization for participating in the syndicate. The concavity of the utility function ensures that a transfer exists that makes both strictly better off. This intuition is formalized in the following claim.

Claim 6. *Suppose that each worker's utility is given by $u_i(c_i, m_i) = \varphi(c_i) + m_i$, where c_i is consumption and m_i is a monetary transfer chosen at the bargaining stage, with $m_1 + m_2 = 0$. Then, revealing the state of the world has no effect on expected welfare.*

Proof. Claims 3 and 4 show that the workers form the syndicate in the absence of a signal, and that this coalition structure maximizes expected welfare. In this case, $m_1 = m_2 = 0$. Next, suppose a signal is drawn. Let $\sigma \in \{(\ell, h), (h, \ell)\}$; without loss of generality, pick $\sigma = (\ell, h)$, the other case being symmetric. Absent side payments, worker 1 would then prefer to form a coalition while worker 2 would not. However, monetary transfers can restore cooperation. Let m denote a transfer from worker 1 to worker 2, with negative values representing transfers in the opposite direction. There always exists an m such that

$$\varphi(H) \leq \varphi\left(\frac{H+L}{2}\right) + m, \quad \varphi(L) \leq \varphi\left(\frac{H+L}{2}\right) - m.$$

To see this, notice that summing these two inequalities yields

$$\varphi(H) + \varphi(L) \leq 2\varphi\left(\frac{H+L}{2}\right),$$

which holds by Jensen's inequality. Hence, syndication is stable when $\sigma \in \{(\ell, h), (h, \ell)\}$. If $\sigma \in \{(\ell, \ell), (h, h)\}$, the workers receive the same utility whether they cooperate or not. Thus, when side payments are allowed, the syndicate forms in exactly those states where cooperation improves welfare, implying that revealing the state has no effect on expected welfare. □

2.3.2. Example: Peer effects

Consider an economy with two *risk-neutral* workers, $N = \{1, 2\}$. Each worker performs an individual task and may experience an idiosyncratic productivity shock. Let $\theta_i \in \{L, H\} \subset \mathbb{R}_+$ denote worker i's productivity level, where $0 < L < H$. We refer to a productivity profile $\theta = (\theta_1, \theta_2) \in \Theta = \{L, H\}^2$ as a state of the world. Productivities are independent across workers, and each worker's productivity is H with probability $p \in (0, 1)$; that is, $\mu(\theta) = p^{k(\theta)}(1-p)^{2-k(\theta)}$ for each $\theta \in \Theta$, where $k(\theta)$ is the number of workers whose productivity is H in state θ. Before the state is realized, the workers decide whether to work in separate rooms or co-locate. In this environment, there are peer effects. If workers operate individually, each produces output equal to her productivity, θ_i, in state θ. If they co-locate, peer effects modify individual output according to relative productivity. Specifically, worker i's output in state θ is

$$q_i(\theta) = \theta_i + \gamma \cdot \mathbf{1}\{\theta_i < \theta_j\} - \delta \cdot \mathbf{1}\{\theta_i > \theta_j\},$$

where $j \neq i$ denotes the other worker and $\gamma, \delta \in (0, 1)$. That is, if exactly one of the two workers experiences the negative productivity shock, the less productive worker enjoys an output gain γ from co-locating with the more productive peer, while the latter incurs an output loss δ. If the workers have identical productivity, co-location has no effect on output. A worker's income and utility equal her output. This game has two possible coalition structures, corresponding to working separately and co-locating, respectively; i.e., $\{\{1\}, \{2\}\}$ and $\{\{1, 2\}\}$. For simplicity, we refer to these coalition structures as no-coop and coop, respectively.

In this example, workers are risk neutral by assumption, so for public information to affect welfare, nonlinearity must arise from primitives other than concave utility. A natural source of such nonlinearity is the peer-effects technology; e.g., $\gamma > \delta$.

Claim 7. *Suppose that $\gamma > \delta$. Then, expected welfare is maximized when the workers co-locate and the only ex-ante stable coalition structure is* coop.

Proof. When working separately, each worker's output equals her productivity; thus, welfare is

$$W(\text{no-coop}) = \sum_{\theta \in \Theta} \mu(\theta)\,(\theta_1 + \theta_2) = 2\,[pH + (1-p)L]\,.$$

Under co-location, peer effects arise when the workers' productivities differ. Specifically, when $\theta_1 = \theta_2 = L$, total output is $2L$; when $\theta_1 = \theta_2 = H$, it is $2H$; and when $\theta_1 \neq \theta_2$, the less productive worker gains γ and the more productive one loses δ. Thus, total output is $H + L + (\gamma - \delta)$ and expected welfare under co-location is

$$W(\text{coop}) = p^2 \cdot 2H + (1-p)^2 \cdot 2L + 2p(1-p) \cdot [H + L + (\gamma - \delta)]\,.$$

Thus,

$$\begin{aligned} W(\text{coop}) - W(\text{no-coop}) &= p^2 \cdot 2H + (1-p)^2 \cdot 2L + 2p(1-p) \cdot [H + L + (\gamma - \delta)] \\ &\quad - 2\,[pH + (1-p)L] \\ &= 2p(1-p)(\gamma - \delta). \end{aligned}$$

Since $\gamma > \delta$, this expression is strictly positive; i.e., co-location maximizes expected welfare.

A worker's expected utility under co-location and working alone is $W(\text{coop})/2$ and $W(\text{no-coop})/2$, respectively. Since $\gamma > \delta$ implies $W(\text{coop}) > W(\text{no-coop})$, working separately yields strictly lower utility. Hence, coop is the only ex-ante stable coalition structure. □

Let us now consider a planner who seeks to maximize expected total output, which coincides with utilitarian welfare in this setting. As before, suppose the planner can draw a perfectly informative public signal about each worker's productivity shock, with realizations $\sigma = (\sigma_1, \sigma_2) \in \{\ell, h\}^2$, before they decide whether to co-locate. The following claim shows that, under the nonlinear peer-effects technology described above, public information reduces expected welfare.

Claim 8. *Suppose that $\gamma > \delta$. Then, drawing a perfectly informative signal about the state of the world decreases expected welfare.*

Proof. For each signal realization σ with $\sigma_1 \neq \sigma_2$, the less productive worker strictly prefers to co-locate, while the more productive worker strictly prefers to work alone; thus, co-location does not occur. From an ex-ante perspective, each worker's expected utility when a signal is drawn is

$$pH + (1-p)L.$$

By Claim 7, this expected utility is strictly lower than the expected utility without a signal. □

This result is not driven by missing risk-sharing opportunities, as the workers are risk neutral. Instead, it reflects the fact that uncertainty helps align individual incentives with socially desirable outcomes. In the absence of side payments, uncertainty sustains cooperation by implicitly offering each worker a favorable expected trade: a large gain when less productive, in exchange for a small loss when more productive. When the state is revealed, this implicit gamble can no longer be implemented, and cooperation unravels.

If side payments are allowed, public information becomes welfare neutral, as in Subsection 2.3.1. Transfers can be used to restore cooperation whenever peer effects make co-location efficient, ensuring that cooperation occurs precisely in the states where it increases total output.

2.3.3. Example: Joint venture

Two entrepreneurs can undertake a project that requires participation from both. Participation entails a monetary cost $\kappa \in (0, 1/2)$ for each entrepreneur. The project yields output $q = \theta$, where the state $\theta \in \Theta = [0, 1]$ is drawn from the uniform distribution on $[0, 1]$ and realized after the participation decisions.[16] Each entrepreneur $i = 1, 2$ has preferences over consumption represented by a vNM utility function $\varphi(c_i) = -\exp(-\alpha c_i)$, with $\alpha > 0$. If the project is undertaken, each entrepreneur receives consumption $c_i = \frac{q}{2} - \kappa$; if not, both consume zero. This game admits two possible coalition structures: $\{\{1\}, \{2\}\}$, corresponding to not undertaking the project, and $\{\{1, 2\}\}$, corresponding to undertaking it. We refer to these structures as no-coop and coop, respectively. Suppose a utilitarian planner can draw a perfectly informative public signal about the project's output before the entrepreneurs decide whether to undertake it. The following claim shows that this signal strictly increases expected welfare.

Claim 9. *A perfectly informative public signal about the state θ strictly increases expected welfare.*

Proof. Without a signal, entrepreneurs undertake the project as long as

$$\int_0^1 \varphi\left(\tfrac{\theta}{2} - \kappa\right) \, \mathrm{d}\theta \geq -1;$$

thus, welfare is

$$2 \max\left\{\int_0^1 \varphi\left(\tfrac{\theta}{2} - \kappa\right) \, \mathrm{d}\theta, \; -1\right\}.$$

When a signal is drawn, the project is undertaken in state θ if and only if

$$\varphi\left(\tfrac{\theta}{2} - \kappa\right) \geq \varphi(0).$$

Thus, expected welfare when a signal is drawn is

$$2\int_0^1 \max\left\{\varphi\left(\tfrac{\theta}{2} - \kappa\right), -1\right\} \, \mathrm{d}\theta > 2 \max\left\{\int_0^1 \varphi\left(\tfrac{\theta}{2} - \kappa\right) \, \mathrm{d}\theta, \; -1\right\}.$$

This inequality is strict because, if the project is undertaken without a signal, participation occurs even in states where $\frac{\theta}{2} - \kappa < 0$. Conversely, if the project is not undertaken without a signal, participation in states where $\frac{\theta}{2} - \kappa > 0$ is forgone. □

2.3.4. Example: Favor exchange

Two individuals are each working on an individual project and have the option to "exchange favors," meaning that each agrees to contribute effort to assist the other's project. If they do not exchange favors, each works independently and receives a utility normalized to zero. If they do, they exert effort to help each

[16] For convenience, this example uses a continuous state space, whereas the model in Section 2 assumes a finite one. All definitions and results extend to general state spaces by replacing sums with integrals and defining posteriors via conditional densities. Alternatively, the example can be approximated by discretizing $[0, 1]$ into finitely many states.

other, but the quality of help is uncertain. A state of the world is a profile $\theta = (\theta_1, \theta_2) \in \Theta = \{L, H\}^2$, where θ_i is the quality of individual i's favor, low (L) or high (H). Qualities are independent across individuals, and each favor is of high quality with probability $p \in (0, 1)$; that is, $\mu(\theta) = p^{k(\theta)}(1-p)^{2-k(\theta)}$ for each $\theta \in \Theta$, where $k(\theta)$ is the number of high-quality favors in state θ. The benefit of favor exchange to individual i is g in states where the partner's favor is of high quality and b in states where it is of low quality, where $b < g$. Regardless of the state, exchanging favors requires each individual to bear a utility cost c. Thus, the expected utility of favor exchange is

$$pg + (1-p)b - c.$$

There are two possible coalition structures: $\{\{1\}, \{2\}\}$, corresponding to not exchanging favors, and $\{\{1, 2\}\}$, corresponding to helping each other. We refer to these structures as no-coop and coop, respectively. Suppose a utilitarian planner can draw a perfectly informative public signal about the quality of help before the individuals decide whether to exchange favors. The following claim shows that if the cost of favor exchange is low or high enough, the social value of this signal is zero.

Claim 10. *If $c < b$ or $c > g$, a perfectly informative signal about the state of the world does not affect expected welfare.*

Proof. If $c < b$, individuals always exchange favors, regardless of whether a signal is observed. If $c > g$, they never exchange favors. In both cases, public information does not affect behavior and therefore has no effect on welfare. □

The interesting case arises when $b < c < g$, in which the social value of public information may be positive or negative. The following claim shows that, for values of c sufficiently close to g, public information improves welfare.

Claim 11. *If $c \in (pg + (1-p)\, b, g)$, drawing a perfectly informative signal about the state of the world strictly increases expected welfare.*

Proof. First of all, notice that if $c \in (b, g)$ and a signal is drawn, individuals cooperate only when both favors are of high quality. As a result, expected welfare is

$$2p^2(g - c).$$

On the other hand, the individuals' behavior without a signal depends on the sign of $pg + (1-p)b - c$. Suppose that

$$c > pg + (1-p)b.$$

Then, the individuals do not exchange favors without a signal. In this case, drawing a signal increases expected welfare because it allows them to cooperate when both favors are of high quality. □

If $c \in (b, p(g-b) + b)$, individuals choose to exchange favors when no signal is drawn. In this range, public information can reduce expected welfare. The reason is that, once help quality is revealed, socially beneficial favor exchange is no longer stable in states where exactly one favor is of high quality.

Without side payments, the individual providing high-quality help has no incentive to cooperate with a partner offering only low-quality help. Public information does help avoid inefficient cooperation when

both favors are of low quality, but overall welfare falls if this gain is outweighed by the loss from deterring cooperation in asymmetric states. This trade-off is formalized in the following claim.

Claim 12. *Suppose $c \in (b, p(g-b)+b)$. Then, drawing a perfectly informative signal about the quality of favors affects welfare as follows:*

- *If*
$$2p(1-p)(g+b-2c) > 2(1-p)^2(c-b),$$
public information decreases expected welfare.

- *If*
$$2p(1-p)(g+b-2c) = 2(1-p)^2(c-b),$$
public information has no effect on expected welfare.

- *If*
$$2p(1-p)(g+b-2c) < 2(1-p)^2(c-b),$$
public information increases expected welfare.

Proof. If $c \in (b, p(g-b)+b)$, individuals find it optimal to exchange favors when no signal is drawn. In this case, expected welfare is

$$2\left[pg+(1-p)b-c\right] = 2\left[p^2(g-c)+p(1-p)(g+b-2c) + (1-p)^2(b-c)\right].$$

If a signal is drawn, individuals cooperate only when both favors are of high quality. In that case, expected welfare is

$$2p^2(g-c).$$

Hence, expected welfare without a signal minus expected welfare with a signal is

$$2p(1-p)(g+b-2c) - 2(1-p)^2(c-b).$$

Public information decreases welfare if this difference is positive, has no effect if it is zero, and increases welfare if it is negative. □

2.4. Further discussion

As explained after Theorem 1, the welfare effect of public information may depend on the selection rule used to evaluate welfare. The results in Subsections 2.2 and 2.3 do not depend on it. Indeed, in transferable utility games, every core allocation yields the same total welfare, and in the examples of Subsection 2.3, all stable coalition structures yield the same welfare whenever more than one is stable.

3. Conclusions

This paper develops a framework for understanding the social value of public information in a general class of cooperative settings. It shows that this value is not fundamentally tied to how information affects

the scope for risk-sharing, but instead reflects a broader tension between efficiency and stability. This perspective clarifies why public information may reduce welfare even in the absence of risk aversion, and connects the literature on the social value of information to classic game-theoretic questions in coalition formation and bargaining.

The social value of public information hinges on whether public signals destabilize socially desirable cooperative arrangements. When public signals do not destabilize any cooperative arrangement that is stable without them, public information cannot reduce welfare. Conversely, when public information reshapes which cooperative arrangements are stable, it can either facilitate or *hinder* the formation of socially desirable cooperative arrangements, making its welfare effect fundamentally ambiguous.

CRediT authorship contribution statement

Caio Lorecchio: Validation, Formal analysis, Writing - Review & Editing.

Davide Pietrobon: Conceptualization, Methodology, Validation, Formal analysis, Writing - Original Draft, Writing - Review & Editing, Visualization.

Declaration of competing interest

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During the preparation of this work the authors used Claude (Anthropic) in order to check mathematical notation and proofs for consistency and to edit the language. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

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